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Investigation on the characteristics of the flow and heat transfer in bilaterally heated narrow annuli

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Abstract

The characteristics of the flow and convective heat transfer on the conditions of both developed laminar and turbulent flow in bilaterally heat narrow annuli were investigated. Investigations show that the ratio of heating flux at the inner wall to that at the outer wall, and the gap size of narrow annuli as well, have great influences on the heat transfer characteristics during flowing in bilaterally heated narrow annuli. Under laminar flow condition, the decreases of the gap size will lead to the heat transfer deterioration both on the inner and outer walls. However with respect to the case of the turbulent flow conditions, it is quite different. The decrease of the gap size will yield heat transfer deterioration on the inner wall, but it will enhance the heat transfer coefficient on the outer wall. In addition, whether the convective heat transfer is enhanced or reduced depends on the combination effect of the heat-flux ratio and the Reynolds number. The decrease of the gap size will reduce the convective heat transfer when the heat-flux ratio is smaller, and enhance the heat transfer coefficient when the heat-flux ratio becomes greater under a certain Reynolds number. Predicted friction coefficients and heat transfer coefficients were compared to experimental data as well, and they were found to be in good agreement.

Keywords: Narrow annular channel; Heat transfer characteristic; Heat-flux ratio; Gap size

1. Introduction

In recent years, increasingly attention has been given to the compact heat exchanger due to its intrinsic superior characteristics of high thermal efficiencies, small size, light weight and energy saving and can be used in various industrial progress, such as cooling of electric device and nuclear reactor. As one kind of this heat transfer tube in compact exchanger is the concentric narrow annuli, and many scholars devoted their efforts to the studies on the flow and convection characteristics of fluid flow in narrow annular channels. Available data suggest important differences of flow and convection characteristics exist between conventional and narrow channels. In addition, existing experimental results on the flow and heat transfer phenomena in narrow annuli show a great deal of discrepancies, and even contrary to one another.

Sun et al. [1], Sun et al. [2] performed series experimental investigation on forced flow and heat transfer with water following in bilaterally heated narrow annular channels with gap size of 1.0 mm and 0.9 mm, respectively. Comparing to unilaterally heating condition, they found that bilaterally heating condition enhanced the heat transfer on the inner surface of an annulus when Reynolds number is higher and reduced the heat transfer on the outer surface when Reynolds number is small. Also using water as circulating fluid, the experimental results of Li et al. [3] with gap size of 1.0 mm showed that the narrow annular channel manifested itself in the effect of enhanced heat transfer in the region of $Re < 2000$. Using fluorine-113 as circulating fluid, Li et al. [4] carried out experimental investigation

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Nomenclature

c_p	specific heat capacity (J kg^{-1} K^{-1})
D_e	equivalent hydraulic diameter $2(R_2 - R_1)$ (m)
f	Darcy friction factor
h	heat transfer coefficient (W m^{-2} K^{-1})
k	thermal conductivity (W m^{-1} K^{-1})
Nu	Nusselt number
p	pressure (Pa)
Pr	Prandtl number
Pr_t	turbulent Prandtl number
q	radial heat-flux density (W m^{-2})
r	radial coordinate (m)
R	radius of a round tube (m) or dimensionless radial coordinate r/R_2
R_0	radial location of zero shear stress (m)
Re	Reynolds number
R_{max}	radial location of maximum velocity (m)
T	time-averaged temperature (K)
T'	fluctuating component of temperature (K)
T^+	dimensionless temperature $k\sqrt{\rho\tau_{W1}}(T_{W1} - T)/(\mu q_{W1})$
$\overline{T'v'}$	time-averaged of product of fluctuating temperature and velocity (K m s^{-1})
$(\overline{T'v'})^+$	local fractional of radial heat-flux density due to turbulence $(\rho c_p \overline{T'v'}/q)$
u	axial component of time-averaged velocity (m s^{-1})
u^+	dimensionless velocity in flow direction $u\sqrt{\rho/\tau_W}$
$\overline{u'v'}$	time-averaged of product of fluctuating components of velocity (m^2 s^{-2})
$\overline{(u'v')}^+$	dimensionless shear stress normalized by τ_W
$\overline{(u'v')}^{++}$	local fractional of shear stress due to turbulence $-\rho\overline{u'v'}/\tau$
v	radial component of time-averaged velocity (m s^{-1})
v'	fluctuating component of radial velocity (m s^{-1})
y	distance from the wall (m)
y^+	dimensionless distance to the wall $y\sqrt{\rho\tau_W}/\mu$
z	axial coordinate (m)
w	velocity in the flowing direction (m s^{-1})
W	dimensionless velocity in Eqs. (1) and (2)

Greek symbols

β	radius ratio, R_1/R_2
δ	gap size of annulus (m)
ζ	dimensionless radial coordinate in Eq. (1)
Θ	dimensionless temperature in Eq. (2)
ν	kinematic viscosity (m^2 s^{-1})
μ	dynamic viscosity (Pa s)
μ_t	eddy dynamic viscosity (Pa s)
ρ	specific density (kg m^{-3})
τ	shear stress (Pa)
τ_m	mean shear stress (Pa)
ϖ	Heat-flux ratio, q_{W1}/q_{W2}

Subscripts

m	mean
1, W1	pertains to the inner wall
2, W2	pertains to the outer wall

on the flow and heat transfer in narrow annular channels with the gap size of 1.0, 1.5 and 2.5 mm. Their experimental results showed that the Nusselt number of the single-phase flow in narrow annuli was higher than that in the common circular pipes. The Nusselt number at the inner wall of the annulus was higher than that at the outer wall. The differences depended on the narrow gap size, and reach its maximum value when the gap size was 1.0 mm. The research group of the present authors, Peng [5,6], and Wu [7] also had conducted series experimental investigation on the flow and heat transfer with distilled water as circulating fluid in narrow annular channels with gap size of 1.0, 1.5 and 2.0 mm, respectively. Their test loop is a closed-loop system including a shield pump, a pressurizer, a venturi flowmeter, a preheater and a condenser. Distilled water was pumped through the loop. The test section is composed of two concentric straight stainless steel tubes, with the coolant flowing through the annular space (narrow gap) between the inner and outer walls. The outer tube and inner tube of the test section are heated with AC by the power supply (low voltage and high current). To reduce the heat loss from the test section, the whole test section was first wrapped by fiberglasses 120 mm in thicknesses, a wire heater was wound outside of this heat insulator to compensate the heat loss, and then another fiberglasses 50 mm in thickness is wrapped outside of the wire heater. More details about the test loop and test section can be found in Refs. [5–7]. Their experimental results showed that different to that of unilaterally heating conditions, the heating flux at the inner wall of an annulus had great impact upon the heat transfer between the outer wall and the circulating fluid, and vice versa. Compared with unilaterally heating conditions, the heat transfer at the inner wall of an annulus will be enhanced while the heat transfer efficient at the outer wall will be reduced slightly under the case of bilaterally heating condition. Meanwhile introducing k–kL–$\overline{uv}$ turbulent model, Liu et al. [8] carried out numerical calculations to predict the heat transfer coefficient in narrow annular channels, and their results showed that the decrease of the annular gap size would yield heat transfer enhancement during flowing in narrow annular channels.

A perusal of the relevant literature indicates that the mechanism of flow and heat transfer in narrow annular channels is far from being understood, and the studies of the thermal interferences between the inner and outer wall of an annulus and their impact upon the convective heat

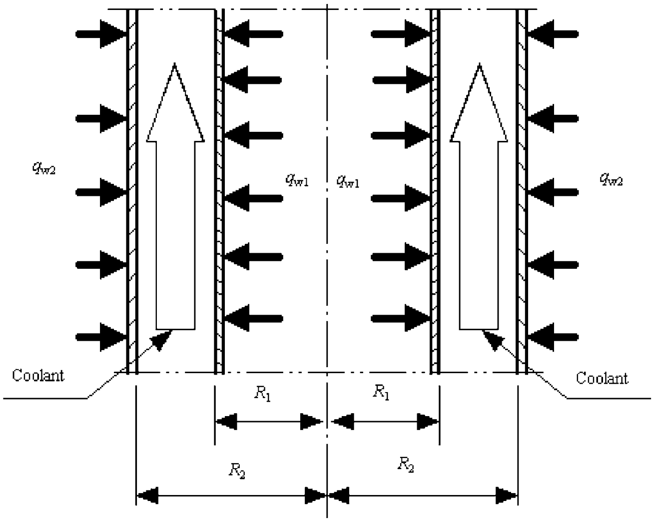

Fig. 1. Schematic diagram of bilaterally heated narrow annuli.

transfer coefficient is very scarce. The present study aimed to this purpose. A schematic diagram of a bilaterally heated narrow annular channel is illustrated in Fig. 1. For simplicity, only the case of fully developed flow, including developed laminar and turbulent flow in narrow annuli were studied. The organization of the rest of this paper is as follows. In Section 2, the effect of the ratio of the heating flux at the inner wall of an annulus to that at the outer wall on the convective heat transfer characteristics for fully developed laminar flow in narrow annuli were studied. Section 3 discussed the fully developed turbulent flow, and compared the predicted friction factor to the experimental data. In Section 4, investigations were carried out on the convective heat transfer characteristics under the case of turbulent flow through narrow annular channels, and good agreement were found between the predicted Nusselt numbers and the experimental values. And finally, this paper is ended with conclusions in Section 5.

2. Heat transfer under laminar flow condition

The heat transfer characteristics of the fully developed laminar flow in narrow annular channels were discussed in this section. As it had been intensively studied in our previous paper [9], which will be briefly introduced for comparison to that under fully developed turbulent flow condition and formation of an overall insight into the mechanisms of the flow and heat transfer characteristics in bilaterally heated narrow annuli.

2.1. Models for developed laminar flow

Without considering the effect of viscous dissipation and axial heat conduction, the conservation equations of momentum and energy for fully developed laminar fluid with invariant physical properties in a concentric narrow annular channel, where the inner and outer walls are heated by uniform but unequal heat fluxes, may be written in the following dimensionless forms:

$$\frac{1}{\zeta}\frac{\partial}{\partial\zeta}\left(\zeta\frac{\partial W}{\partial\zeta}\right)+\frac{1}{\zeta}\frac{\partial}{\partial\theta}\left(\frac{1}{\zeta}\frac{\partial W}{\partial\theta}\right)+1=0 \tag{1}$$

$$\frac{1}{\zeta}\frac{\partial}{\partial\zeta}\left(\zeta\frac{\partial\Theta}{\partial\zeta}\right)+\frac{1}{\zeta}\frac{\partial}{\partial\theta}\left(\frac{1}{\zeta}\frac{\partial\Theta}{\partial\theta}\right)=\frac{W}{W_{\mathrm{m}}}\frac{2}{1-\beta^2} \tag{2}$$

where $\zeta = r/R_2$, $W = -\nu w/(R_2^2\,\mathrm{d}p/\mathrm{d}z)$, $\Theta = (T - T_{\mathrm{m}})/q_{\mathrm{e}}R_2/k$, and $q_{\mathrm{e}} = q_{\mathrm{W2}} + q_{\mathrm{W1}}\beta$ can be considered as the equivalent heating flux on the inner surface of the outer tube, here $\beta = R_1/R_2$ is the diameter ratio of the annular channel.

The coupling conditions of Eqs. (1) and (2) are as follows:

(a) On the inner wall: $\zeta=\beta$, $W=0$ and $\partial\Theta/\partial\varsigma = \varpi/(1+\varpi\beta)$.
(b) On the outer wall: $\zeta=1$, $W=0$ and $\partial\Theta/\partial\varsigma = 1/(1+\varpi\beta)$.
(c) On symmetric line: $\partial W/\partial\theta=0$ and $\partial\Theta/\partial\theta=0$.

According to the definition of dimensionless temperature, the following coupling condition to the energy balance equation can be obtained as another definite condition:

$$\Theta_{\mathrm{m}} \equiv \frac{\int_A \Theta W\,\mathrm{d}A}{\int_A W\,\mathrm{d}A} = \frac{T_{\mathrm{m}}-T_{\mathrm{m}}}{q_{\mathrm{e}}R_2/k} = 0 \tag{3}$$

where A denotes the cross section area of the narrow annular channel.

The finite volume method was adopted to discretize Eqs. (1) and (2). As no significant discrepancy were found with refinement of the grid system, a competent grid system of 22 (in ζ) $\times$ 15 (in θ) were used in each module. When the maximum relative variations of dimensionless temperature and dimensionless velocity in successive two iterations of all the nodes in the computation domain, and the fluid mean dimensionless temperature are less than 1.0×10^{-4}, convergence solutions were assumed to be achieved and calculation were terminated [9]. The radial dimensionless temperature and dimensionless velocity distribution can be obtained, and then, the heat transfer coefficients both at the inner and outer surface of the narrow annuli can be obtained as follows.

For the outer surface of the inner tube

$$Nu_1 \equiv \frac{q_{\mathrm{W1}}}{T_{\mathrm{W1}}-T_{\mathrm{m}}}\frac{D_{\mathrm{e}}}{k} = \frac{D_{\mathrm{e}}q_{\mathrm{W1}}}{\Theta_{\mathrm{W1}}R_2q_{\mathrm{e}}} = \frac{D_{\mathrm{e}}}{\Theta_{\mathrm{W1}}R_2}\frac{\varpi}{1+\beta\varpi} \tag{4}$$

For the inner surface of the outer tube

$$Nu_2 \equiv \frac{q_{\mathrm{W2}}}{T_{\mathrm{W2}}-T_{\mathrm{m}}}\frac{D_{\mathrm{e}}}{k} = \frac{D_{\mathrm{e}}q_{\mathrm{W2}}}{\Theta_{\mathrm{W2}}q_{\mathrm{e}}R_2} = \frac{D_{\mathrm{e}}}{\Theta_{\mathrm{W2}}R_2}\frac{1}{1+\beta\varpi} \tag{5}$$

2.2. Results under developed laminar flow condition

The following results were based on the reference narrow annuli, and their geometric parameters are listed in Table 1.

Fig. 2 shows the effect of the ratio of heating flux at the inner surface to that at the outer surface of an annulus on the radial temperature distribution in narrow annular channels. For the sake of the limitation of the paper length, and the same characteristics were found in other gap sizes, herein only the results of the gap size 2.0 mm were shown.

Table 1
Geometric parameters of reference narrow annuli

Inner radius R_1 (mm)	Outer radius R_2 (mm)	Gap size δ (mm)	Radius aspect β
4.0	5.0	1.0	0.8
3.5	5.0	1.5	0.7
3.0	5.0	2.0	0.6

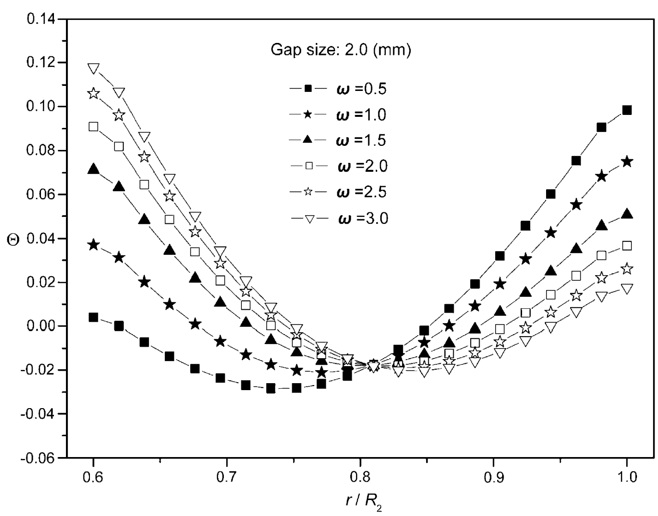

Fig. 2. The influence of the heat-flux ratio on the radial temperature profile.

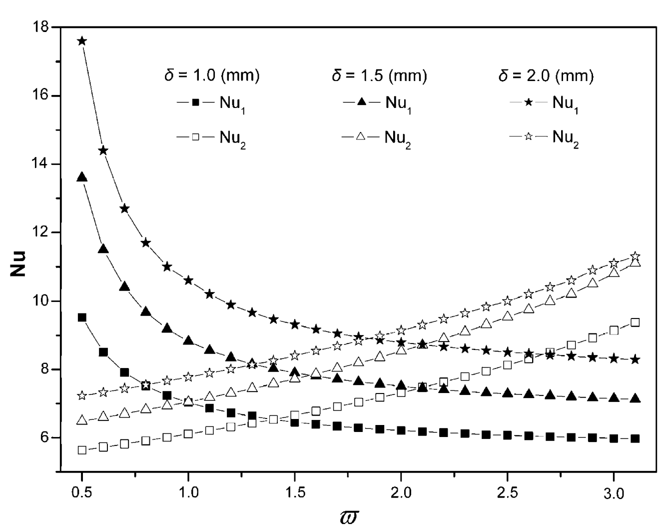

Fig. 3. The influence of the heat-flux ratio on the heat transfer coefficient.

Fig. 2 shows clearly that the heat-flux ratio has great impact upon the radial temperature profile, then, upon the heat transfer. From Fig. 3, it can be seen clearly that Nusselt number at the outer surface of the inner tube decreases with the increase of the heat-flux ratio of the inner surface to outer surface, while the Nusselt number at the inner surface of the outer tube increases with the increase of the heat-flux ratio. Addition to that, the Nusselt number at the inner surface is greater than that at the outer surface when the heat-flux ratio is smaller than some value, for example about 1.5 for the case of 1.0 mm in annular gap size, but it become smaller than the Nusselt number at the outer surface when the heat-flux ratio is greater than about 1.5 for the same case. Fig. 3 also shows clearly that the decrease of the annular gap size may yield heat transfer deterioration during flowing in narrow annular channels.

3. Flow characteristics under turbulent flow condition

The characteristics of flow and heat transfer during flow in conventional concentric circular annuli were widely investigated both experimentally and theoretically. The main concern in the earlier experimental studies was the coincidence of the radial positions between zero shear stress and the maximum velocity. Rehme [10] observed the noncoincidence of the positions between zero shear stress and maximum velocity through an intensive experimental work. More over, he found that the position of zero shear stress is closer to the inner wall than that of maximum velocity. The concept of eddy viscosity is found to be unbounded at the location of maximum velocity and negative over an adjacent region near the location of zero shear stress. Obviously, the eddy viscosity concept is not truly applicable for an annulus. Semiempirical differential equations of conservation for the kinetic energy of turbulence and the rate of dissipation of turbulence, which functions by generating the eddy viscosity and were very geometry-dependent for introducing many variants and modifications, is also precluded for an annulus. The concept of mixing length is subject to similar anomalies. Direct numerical simulation of turbulent flow and convection constitute a new and invaluable resource but are as yet limited to only a few geometries and conditions, because of the truly great computational requirements. To avoiding aforementioned shortcomings, Churchill and Chan [11,12], Churchill [13,14] devised correlation equations for the local time-averaged velocity and the local time-averaged turbulent shear stress in round tube and parallel-plate channels. These correlation equations incorporated all of the known theoretical structure, such as asymptotic behavior near the wall, the asymptotic behavior near the centerline, and the speculative intermediate behavior. Following that procedure, Kaneda et al. [15] proposed modifications to these correlation equations based on recent more accurate result of experiment and direct numerical simulation, and used them to predict the flow and convection in conventional annulus with different aspect ratios.

3.1. Models for developed turbulent flow

The time-averaged equation of momentum for fully developed turbulent flow of a fluid with invariant physical properties in narrow annuli can be expressed in the following forms:

$$-\frac{dP}{dz}+\frac{1}{r}\frac{d}{dr}\left(\mu r\frac{du}{dr}-\rho r\overline{u'v'}\right)=0 \quad (6)$$

where the superbars designating time-averaged values of P and u have been dropped for simplicity, but that for the product of the fluctuating component of the velocity is retained for clarity.

Based on Eq. (6), the time-averaged velocity in the direction of fluid flow can be obtained as

$$u=\frac{1}{4\mu}\left(-\frac{dP}{dz}\right)\left[R_2^2-r^2-\left(R_2^2-R_1^2\right)\frac{\ln(R_2/r)}{\ln(R_2/R_1)}\right]+\frac{\rho}{\mu}\cdot\frac{\ln(R_2/r)}{\ln(R_2/R_1)}\int_{R_1}^{R_2}\overline{u'v'}\,dr-\frac{\rho}{\mu}\cdot\int_{r}^{R_2}\overline{u'v'}\,dr \quad (7)$$

and

$$u_m=\frac{1}{8\mu}\left(-\frac{dP}{dz}\right)\left[R_2^2+R_1^2-\frac{R_2^2-R_1^2}{\ln(R_2/R_1)}\right]+\frac{\rho}{2\mu\ln(R_2/R_1)}\int_{R_1}^{R_2}\overline{u'v'}\,dr-\frac{\rho}{\mu(R_2^2-R_1^2)}\int_{R_1}^{R_2}\overline{u'v'}r^2\,dr \quad (8)$$

Introducing the definition of the mean shear stress over the two walls: $\tau_m=(R_1\tau_{W1}+R_2\tau_{W2})/(R_1+R_2)$, which is related to the pressure gradient according to the overall force balance as $\tau_m=(-dP/dz)(R_2-R_1)$, the definition of friction coefficient $(-dP/dz)=f/D_e\cdot\rho u_m^2/2$, and the Reynolds number $Re=D_e\rho u_m/\mu$, Eq. (8) may be rewritten as the following dimensionless form:

$$\frac{1}{f\cdot Re}=\frac{1-\beta^2+(1+\beta^2)\ln\beta}{64(1-\beta)^2\ln\beta}+\frac{(1+\beta)\int_{R_1}^{R_2}\overline{(u'v')}^+\,d(r/R_2)}{16(1-\beta)(\beta+\tau_{W2}/\tau_{W1})\ln\beta}-\frac{\int_{R_1}^{R_2}\overline{(u'v')}^+(r/R_2)^2\,d(r/R_2)}{8(1-\beta)^2(\beta+\tau_{W2}/\tau_{W1})} \quad (9)$$

The quantity $\overline{(u'v')}^+$ may vanish for laminar flow and Eq. (9) can be expressed as

$$\frac{1}{f\cdot Re}=\frac{1+\beta^2+(1-\beta^2)/\ln\beta}{64(1-\beta)^2} \quad (10)$$

Churchill and Chan [11] developed the correlation equations for the local time-averaged turbulent shear stress in smooth round tubes and parallel-plate channels. These correlation equations were presumed to be applicable for the prediction of the fully developed turbulent flow in concentric annuli, where the flow field was subdivided into two radial regions delimited by the maximum velocity location [15]. However, separate expressions were devised for the inner region, $R_1<r<R_{max}$, and the outer region, $R_{max}<r<R_2$. Both of them have the overall form [11,15]

$$[\overline{(u'v')^+}]^n=[\overline{(u'v')}_0^+]^n+[\overline{(u'v')}_\infty^+]^n \quad (11)$$

where the mean value of the exponent n is equal to $-8/7$.

On the basis of direct numerical simulations for parallel-plate channels, the asymptotic expression for small values of y^+ is presumed to be applicable for all shear flows, including the inner and outer regions of an annulus [11]:

$$\overline{(u'v')}_0^+=0.7(y^+/10)^3 \quad (12)$$

For correlating $\overline{(u'v')}_\infty^+$, the following dimensionless form for the time-mean velocity distribution in the turbulent core of a round tube was devised by Churchill and Chan [11]:

$$u^+=A+B\ln(y^+)+C(y^+/R^+)^2-(B+2C)/3\cdot(y^+/R^+)^3 \quad (13)$$

where R denotes the radius of the tube, the constants of A, B and C, according to the recent most accurate experimental data for velocity distribution in a round tube were 6.13, 1/0.436, and 6.824, respectively. These same values are presumed to be directly applicable for the outer region of an annulus, while for the inner region, a different value of C is proposed in order to force the expression for the velocity distribution in the inner and outer regions to converge at $r=R_{max}$ [15]. The combination of the time-averaged force–momentum balance for fully developed turbulent flow and Eq. (13) leads to the following equation for $\overline{(u'v')}_\infty^+$:

$$\overline{(u'v')}_\infty^+=\frac{R_1}{r}\frac{R_0^2-r^2}{R_0^2-R_1^2}-\frac{du^+}{dy^+} \quad (14)$$

It should be noted however that when Eqs. (11)–(14) are used for the inner and outer regions of an annulus, the dimensionless quantities are normalized with respect to inner wall shear stress τ_{W1} and the outer wall shear stress τ_{W2}, respectively, and the relationship of these two shear stresses can be obtained by the means of the force balance as follows:

$$\frac{\tau_{W1}}{\tau_{W2}}=\frac{R_2}{R_1}\left(\frac{R_0^2-R_1^2}{R_2^2-R_0^2}\right) \quad (15)$$

Based on intensive experimental work, Kays and Leung [16] proposed the following correlation equation to predict the location of maximum velocity in concentric annular channels:

$$\frac{R_{max}-R_1}{R_2-R_{max}}=\left(\frac{R_1}{R_2}\right)^{0.343} \quad (16)$$

And to predict the location of zero turbulent shear stress, Rehme's experimental correlation [10] may be used:

$$\frac{R_0 - R_1}{R_2 - R_0} = \left(\frac{R_1}{R_2}\right)^{0.386} \tag{17}$$

3.2. Developed turbulent flow characteristics

To validate the previous described mathematical descriptions for developed turbulent flow, numerical calculation were conducted based on the reference narrow annuli as listed in Table 1. The calculated values of friction coefficients were compared with the experimental data. Fig. 4 shows the radial profiles of dimensionless velocity and the local turbulent shear stress in narrow annuli, and Fig. 5 show the predicted values of friction coefficients with experimental data [6,7]. It can be seen clearly that the local turbulent shear stress in annular channels is nonlinear, strongly depended on r, especially in the near wall regions. The results show that the predicted values were in good agreement with the experimental data, and thus these correlations can be used to predict the heat transfer in the following section.

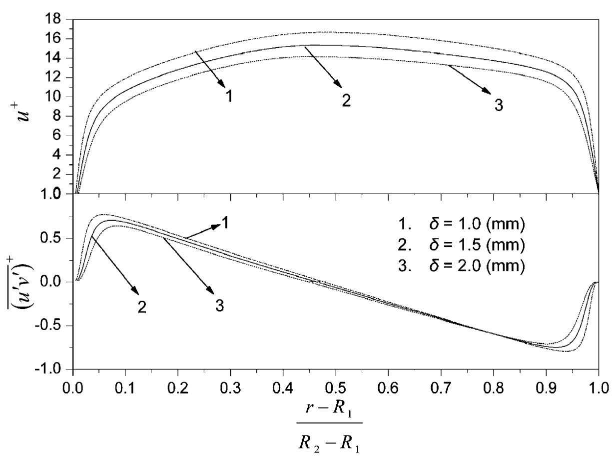

Fig. 4. The turbulent shear stress and the velocity distributions in narrow annuli.

4. Heat transfer characteristics in developed turbulent flow

4.1. Mathematical descriptions for developed turbulent heat transfer

On one hand, the time-averaged partial differential equation for the conservation of energy in the radial direction in fully developed convection with negligible viscous dissipation and axial heat conduction in the fully developed turbulent flow in narrow annuli of a fluid with invariant physical properties may be combined with an overall energy balance to obtain the following dimensionless form as [9,17]

$$\frac{q}{q_{W1}}[1 - (\overline{T'v'})^+] = \frac{dT^+}{dr^+} \tag{18}$$

Instead of correlating the quantity $(\overline{T'v'})^+$ based on experimental data as that of $(\overline{uv'})^+$ in predicting flow, a quite different and much better procedure was substituting the Pr_t/Pr for $(\overline{T'v'})^+$ as the explicit dependence variable. By means of a series steps, the following equation can be obtained [9]

$$\frac{q}{q_{W1}} = \left[1 + \left(\frac{Pr}{Pr_t}\right)\frac{\overline{(u'v')}^{++}}{1 - \overline{(u'v')}^{++}}\right]\frac{dT^+}{dr^+} \tag{19}$$

where the quantity $(\overline{uv'})^{++} = -\rho\overline{u'v'}/\tau$, and based on numerically modified empirical correlation of Jischa and Rieke [18], $Pr_t = 0.85 + 0.015/Pr$ may be used.

Eliminate dT^+/dr^+ between Eqs. (18) and (19), and substitute the quantity $(\overline{uv'})^+$ for $(\overline{uv'})^{++}$ in order to avoid the singular behavior of the quantity $(\overline{uv'})^{++}$ in annulus, results in

$$\frac{Pr_t}{Pr} = \frac{\overline{(u'v')}^+(1 - \overline{(T'v')}^+)}{\overline{(T'v')}^+(\tau/\tau_{W1} - \overline{(u'v')}^+)} \tag{20}$$

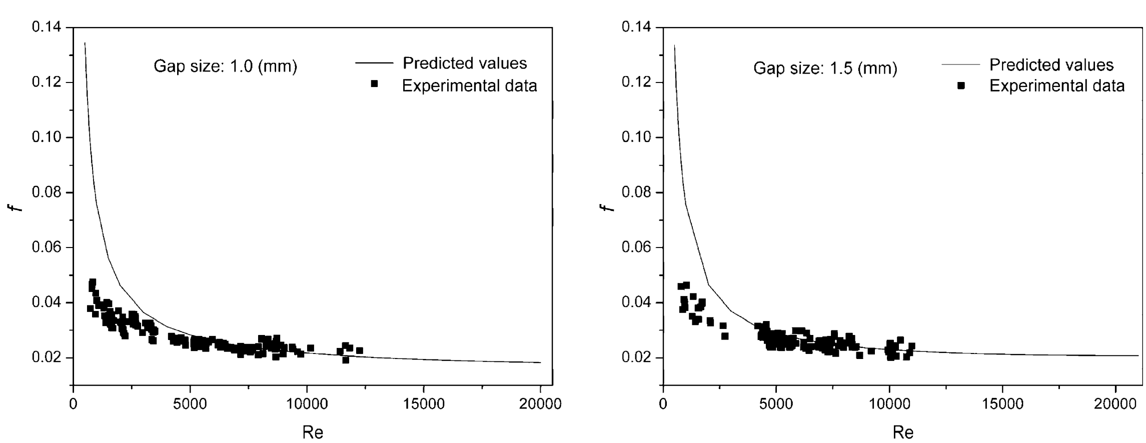

Fig. 5. The comparison of predicted friction coefficients with experimental data.

Substituting for $\left(\overline{T'v'}\right)^{+}$ from Eqs. (18) and (20) yields

$$\frac{q}{q_{\mathrm{W1}}}=\frac{1}{R_{2}^{+}}\left[1+\left(\frac{Pr}{Pr_{\mathrm{t}}}\right)\left(\frac{\overline{(u'v')}^{+}}{\tau/\tau_{\mathrm{W1}}-\overline{(u'v')}^{+}}\right)\right]\frac{\mathrm{d}T^{+}}{\mathrm{d}R} \tag{21}$$

On the other hand, energy balance over an annular segment of fluid between any radial location r and the outer radius of the annulus R_2, the following equation may be obtained:

$$2rq+2R_{2}q_{\mathrm{W2}}=2\rho c_{p}\int_{r}^{R_{2}}ur\frac{\partial T}{\partial z}\,\mathrm{d}z \tag{22}$$

And energy balance over an annular segment of fluid between the inner radius R_1 and the outer radius R_2 yields

$$2R_{1}q_{\mathrm{W1}}+2R_{2}q_{\mathrm{W2}}=\rho u_{\mathrm{m}}c_{p}\left(R_{2}^{2}-R_{1}^{2}\right)\frac{\mathrm{d}T_{\mathrm{m}}}{\mathrm{d}z} \tag{23}$$

Combination of Eqs. (22) and (23) results in

$$\frac{q}{q_{\mathrm{W1}}}=-\frac{1}{R\varpi}+\frac{2(1+\varpi\beta)}{R\varpi(1-\beta^{2})}\int_{R}^{1}\frac{u}{u_{\mathrm{m}}}\frac{\partial T/\partial z}{\mathrm{d}T_{\mathrm{m}}/\mathrm{d}z}R\,\mathrm{d}R \tag{24}$$

For the inner and outer walls of the annulus are heated by uniform heat fluxes, it may be presumed that $\partial T/\partial z=\mathrm{d}T_{\mathrm{m}}/\mathrm{d}z$ Eq. (24) then reduces to

$$\frac{q}{q_{\mathrm{W1}}}=-\frac{1}{R\varpi}+\frac{2(\beta\varpi+1)}{R\varpi(1-\beta^{2})}\int_{R}^{1}\frac{u}{u_{\mathrm{m}}}R\,\mathrm{d}R \tag{25}$$

According to Eqs. (21) and (25), the radial temperature profile, and then the mean temperature of fluid can be worked out by integral method. Finally, Nusselt numbers between fluid to both the inner and outer walls of the annular channels can be determined, that is

For inner wall

$$Nu_{1}\equiv\frac{q_{\mathrm{W1}}}{T_{\mathrm{W1}}-T_{\mathrm{m}}}\cdot\frac{D_{\mathrm{e}}}{k}=\frac{D_{\mathrm{e}}^{+}}{T_{\mathrm{m}}^{+}} \tag{26}$$

For outer wall

$$Nu_{2}\equiv\frac{q_{\mathrm{W2}}}{T_{\mathrm{W2}}-T_{\mathrm{m}}}\cdot\frac{D_{\mathrm{e}}}{k}=\frac{D_{\mathrm{e}}^{+}}{\left(T_{\mathrm{m}}^{+}-T_{\mathrm{W2}}^{+}\right)\varpi} \tag{27}$$

And for the mean Nusselt number of an annulus,

$$\begin{aligned}h_{\mathrm{m}}&\equiv\frac{h_{1}2\pi R_{1}(T_{\mathrm{W1}}-T_{\mathrm{m}})+h_{2}2\pi R_{2}(T_{\mathrm{W2}}-T_{\mathrm{m}})}{2\pi(R_{1}+R_{2})[(T_{\mathrm{W1}}+T_{\mathrm{W2}})/2-T_{\mathrm{m}}]}\\ Nu_{\mathrm{m}}&\equiv\frac{h_{\mathrm{m}}D_{\mathrm{e}}}{k}=\frac{\beta T_{\mathrm{m}}^{+}Nu_{1}+\left(T_{\mathrm{m}}^{+}-T_{\mathrm{W2}}^{+}\right)Nu_{2}}{(1+\beta)\left(T_{\mathrm{m}}^{+}-T_{\mathrm{W2}}^{+}/2\right)}\end{aligned} \tag{28}$$

4.2. Heat transfer evaluation

To validate the aforementioned models, numerical calculations were performed to predict the Nusselt numbers

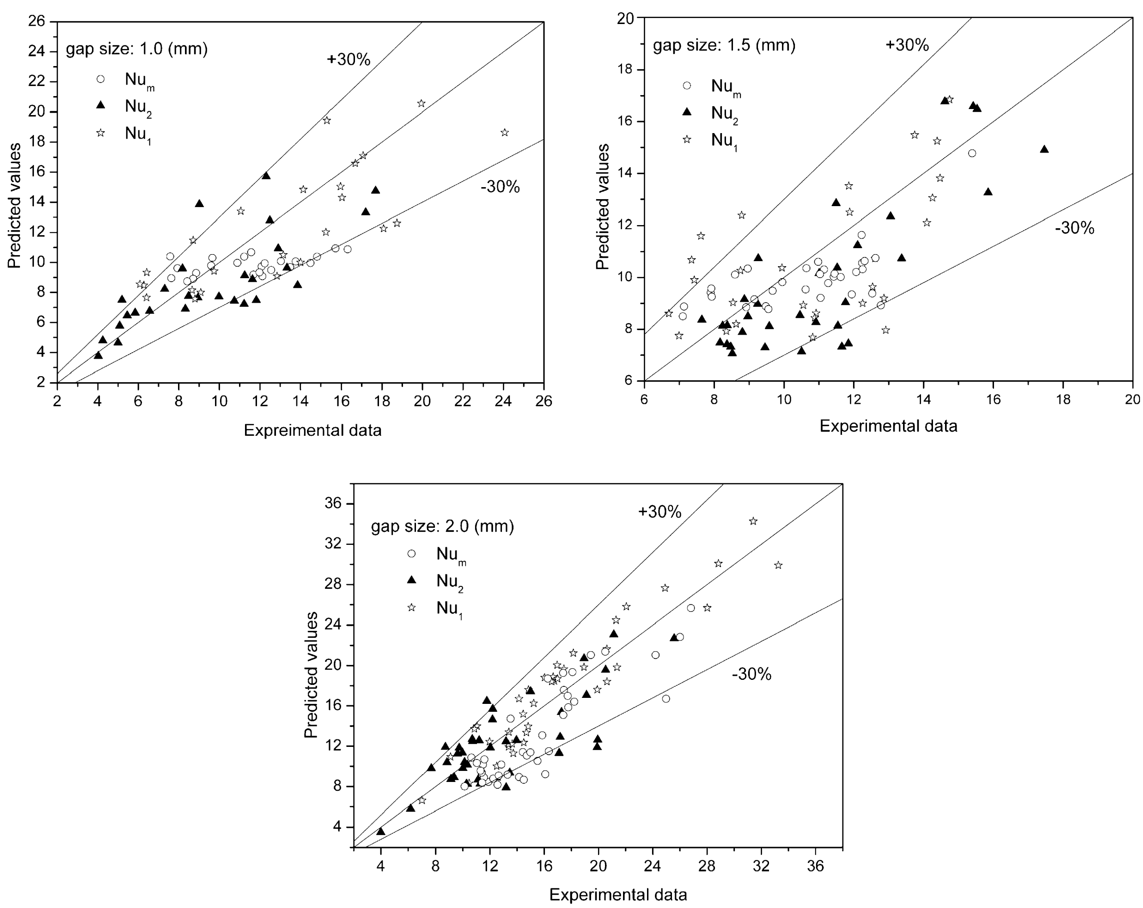

Fig. 6. The comparison of predicted Nusselt numbers with experimental data.

of narrow annuli based on these models, and the predicted results were compared with the experimental data. The geometric parameters of the test sections were listed in Table 1, and the results were plotted in Fig. 6, including experimental data with different gap size of narrow annular channels. Fig. 6 clearly show that the predicted Nusselt numbers of inner wall, Nu_1, outer wall, Nu_2, and averaged ones of an annulus, Nu_m, respectively, were all in good agreement with the experimental data, considering the uncertainties of measurements. These models, as described in the pervious sections, were confirmed by these results, and then could be used to predict the convective heat transfer characteristics in narrow annular channels.

4.3. Discussions of the turbulent convective heat transfer

Based on the aforementioned models, numerical calculations were conducted with respect to the reference narrow annuli as listed in Table 1, and the calculated results were plotted in Fig. 7. Clearly, Fig. 7 shows the influence of the ratio of the heating flux at outer surface of the inner tube to that of the inner surface of the outer tube in narrow annuli. Similar to that under developed laminar flow condition, from Fig. 7 it can be seen clearly that the heat-flux ratio has great impact upon the heat transfer characteristics of the developed turbulent flow in bilaterally heat narrow annular channels. With the increase of the heat-flux ratio, the Nusselt number at the inner surface of an annulus will decrease and leads enhancement to the Nusselt number at the outer wall. In addition, when the heat-flux ratio is small, the Nusselt numbers at the inner wall were greater than that of the outer wall. However, when the heat-flux ratio become greater than a certain value, which depends on the gap size of an annulus and the Reynolds number, the heat transfer coefficient between the outer wall of the annulus and the fluid will surpass the heat transfer between the inner wall of the annulus and the fluid. And the effects of the gap size upon the heat transfer characteristics in narrow annuli can also be seen clearly from Fig. 7, however, the effects of the gap size on the Nusselt numbers are quite different to that under developed laminar flow conditions. Under developed laminar flow condition, the decrease of the gap size leads to heat transfer deterioration both on the inner and outer walls of an annulus as shown in Fig. 3. Under the developed turbulent flow conditions, however, the decrease of the gap size will yields heat transfer deterioration on the inner wall of an annulus, but leads to heat transfer enhancement on the outer wall. When the heat-flux ratio is smaller than, say about 1.4 for the gap size of 2.0 mm and the Reynolds number of 10,000, as shown in

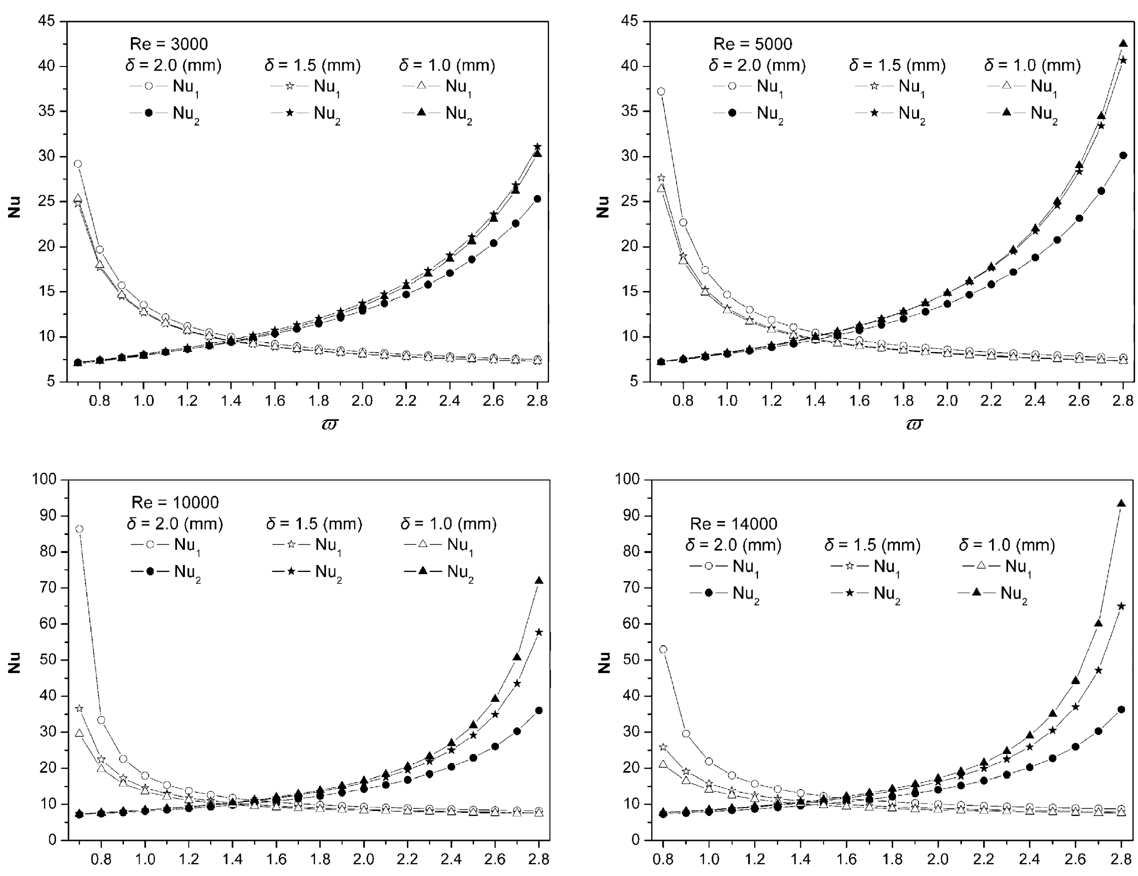

Fig. 7. The influence of heat-flux ratio on the heat transfer under turbulent flow condition.

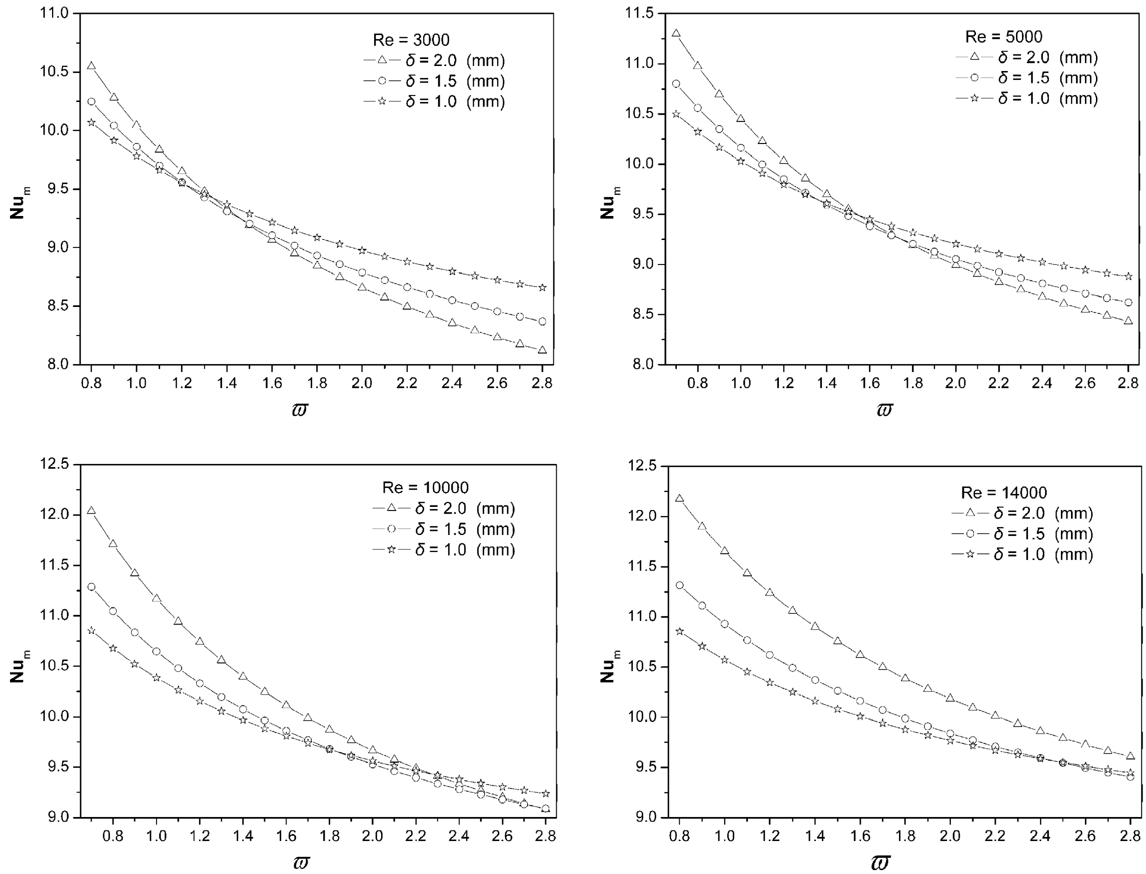

Fig. 8. The effect of heat-flux ratio upon the mean heat transfer coefficients.

Fig. 7, the gap size effect on the outer wall will become negligible. While when the heat-flux ratio become greater, the effect of gap size upon the heat transfer at the inner wall will vanish.

The effects of the heat-flux ratio on the mean convective heat transfer characteristics in narrow annuli were shown in Fig. 8. From Fig. 8 it can be seen clearly that with the increase of the heat-flux ratio, the mean heat transfer coefficient of the narrow annuli will decrease. The effects of heat-flux ratio depend on the gap size of the narrow annuli and the heat-flux ratio. With greater gap size, the mean heat transfer coefficient decreases faster than that with smaller ones. Additional information can also be clearly obtained from Fig. 8. Whether the decrease of the gap size yields the heat transfer enhancement or not also strongly depends on the ratio of heating flux at the inner surface of an annulus to that at the outer surface, and on the flow condition as well, i.e. the Reynolds number. Under the flow condition of lower Reynolds number, when the heat-flux ratio is small, the decrease of the gap size will yield heat transfer deterioration during flowing in narrow annular channels, while when the heat-flux ratio becomes greater than some certain values, which also depend on the Reynolds number, the decrease of the gap size will yield heat transfer enhancement during flowing in narrow annular channels. It may be inferred from these results that as the Reynolds number decrease to some lower values, the decrease of the gap size of a narrow annular channel will only reduce the heat transfer during flowing in narrow annuli, the same tendencies to those under the case of laminar flow conditions.

5. Conclusions

The characteristics of the flow and convective heat transfer on the conditions of both developed laminar and turbulent flow in narrow annular channels were investigated, and the predicted friction coefficients were found to be in good agreement with experimental data. Investigations in the present studies reveal that the parameter, the ratio of heating flux at the inner surface of a narrow annuli to that at the outer surface, and the gap size of narrow annuli as well, have great influences on the heat transfer characteristics during flowing in bilaterally heated narrow annuli, which may be summarized as follows.

Under laminar flow conditions, the decreases of the gap size will always lead to the heat transfer deterioration both on the inner and outer walls during flowing in bilaterally heated narrow annular channels. While the case are quite different when the laminar flow is transferred to turbulent flow. The decrease of the gap size will yields heat transfer deterioration on the inner wall, but it will enhance the heat transfer coefficient on the outer wall of an annulus. In addition, whether the convective heat transfer is enhanced or reduced depends on the combination effect of the heat-flux ratio and the Reynolds number, however the influences of the heat-flux ratio are prominent. The decrease of the gap size will reduce the convective heat transfer when the heat-flux ratio is smaller, and enhance the heat transfer coefficient when the heat-flux ratio becomes greater, under the case of a certain Reynolds number.

References

[1] Z.N. Sun, C.Q. Yan, H.P. Tan, J.Q. Guo, L.C. Sun, Forced convection heat transfer in narrow annulus with bilateral heating, Nucl. Des. Eng. 23 (4) (2002) 33–36.
[2] Z.N. Sun, C.Q. Yan, H.P. Tan, Experimental investigation on forced convection heat transfer through narrow annulus, Nucl. Des. Eng. 24 (4) (2003) 350–353.
[3] Z.J. Li, C.Q. Yan, Z.N. Sun, Experimental investigation on the single-phase heat transfer of casing pipe used for once-through steam generator, J. Harbin Eng. Univ. 21 (5) (2000) 322–325.
[4] B. Li, A.D. He, D.F. Zhou, An experimental investigation on the flow and heat transfer of the narrow-gap tube vapor generators. 1. Single-phase forced flow heat exchange, Chem. Eng. Mach. 28 (1) (2001) 1–4.
[5] C.H. Peng, M. Aye, Y. Guo, D.N. Jia, S.Z. Qiu, G.H. Su, C.H. Nie, Experimental study of flow and heat transfer of narrow annuli with bilaterally heating, Nucl. Power Eng. 24 (suppl. 6) (2003) 92–96.
[6] C.H. Peng, Experimental study of two-phase flow and boiling heat transfer of water in the vertical narrow annuli, Doctor's degree dissertation, School of Energy and Power Engineering, Xi'an Jiaotong University, China, 2005.
[7] G. Wu, Experimental study on heat transfer and flow characteristics of single-phase water and superheated vapor in narrow annular channel, Mater's degree dissertation, School of Energy and Power Engineering, Xi'an Jiaotong University, China, 2005.
[8] R.L. Liu, Z.H. Wang, J.L. Gou, D.N. Jia, Numerical calculation of single phase convective heat transfer in narrow annular channel, Chin. J. Nucl. Sci. Eng. 22 (3) (2002) 229–234.
[9] H.Y. Zeng, S.Z. Qiu, D.N. Jia, Effect of heat-flux ratio on the heat transfer characteristics of fully developed flow in bilaterally heated narrow annuli, J. Xi'an Jiaotong Univ. 40 (3) (2006) 307–310.
[10] K. Rehme, Turbulent flow in smooth concentric annuli with small radius ratios, J. Fluid Mech. 64 (1974) 263–287.
[11] S.W. Churchill, C. Chan, Theoretically based correlating equations for the local characteristics of fully turbulent flow in round tubes and between parallel plates, Ind. Eng. Chem. Res. 34 (4) (1995) 1332–1341.
[12] S.W. Churchill, C. Chan, Turbulent flow in channels in terms of local turbulent shear and normal stresses, AIChE J. 41 (12) (1995) 2513–2521.
[13] S.W. Churchill, New simplified models and formulations for turbulent flow and convection, AIChE J. 43 (5) (1997) 1125–1140.
[14] S.W. Churchill, A critique of predictive and correlative models for turbulent flow and convection, Ind. Eng. Chem. Res. 35 (9) (1996) 3122–3140.
[15] M. Kaneda, B. Yu, H. Ozoe, S.W. Churchill, The characteristics of turbulent flow and convection in concentric circular annuli. Part I: flow, Int. J. Heat Mass Transfer 46 (26) (2003) 5045–5057.
[16] W.M. Kays, K.T. Leung, Heat transfer in annular passages-hydrodynamically developed turbulent flow with arbitrarily prescribed heat flux, Int. J. Heat Mass Transfer 6 (1963) 537–557.
[17] B. Yu, Y. Kawaguchi, M. Kaneda, H. Ozoe, S.W. Churchill, The computed characteristics of turbulent flow and convection in concentric circular annuli. Part II. Uniform heating on the inner surface, Int. J. Heat Mass Transfer 48 (3–4) (2005) 621–634.
[18] M. Jischa, H.B. Rieke, About the prediction of turbulent Prandtl and Schmidt numbers from modified transport equations, Int. J. Heat Mass Transfer 22 (1979) 1547–1555.